

StatR Math Skills Self-Assessment

To succeed in the StatR Certificate, you should be familiar with the topics covered here. If you cannot answer some or many of these questions, then we ask that you brush up on these topics before the program begins. We can help identify resources for practice, if necessary.

You **do not** need to submit your answers to UW Professional and Continuing Education, but you must confirm that you are comfortable with the level of math skills in these questions, and that you understand that you are expected to know this level of math skills if you are accepted into and start the Certificate.

Logarithms and Algebra

The definition of a logarithm is that $\log_b x = y$ is equivalent to $b^y = x$ for any $x > 0$, $b > 0$ and $b \neq 1$. We will use the computer science convention (different from the mathematical convention) and assume that the base of a logarithm is e unless otherwise specified.

1. What is the value of $\log_2 32$?
2. What is the value of $\log_3 \frac{1}{27}$?
3. If $y = e^{3x-1}$, what is the slope and intercept of $\log(y)$ with respect to x ?
4. If $y = 4e^{-x}$, what is the slope and intercept of $\log(y)$ with respect to x ?
5. What is $(a+b)^2 - (a^2 + b^2)$?
6. If $f(x) = 3x^2 - 2$ and $g(x) = x + 2$, what is $f(g(0))$?

Calculus

1. Describe a process for finding the maximum of a function $f(x)$.
2. Let $f(x) = 3e^{2x+1}$. Find $f'(x)$.
3. Let $f(x) = \log(x^4 \cdot (1-x)^6)$. Find $f'(x)$. (*Hint*: simplify before differentiating.)
4. If $f(x) \geq 0$, $f(-x) = f(x)$, and $\int_{-\infty}^{\infty} f(x)dx = 1$, what is $\int_{-\infty}^0 f(x)dx$?
5. If $f(x) \geq 0$ and $f(-x) = f(x)$, what is $\int_{-\infty}^{\infty} x \cdot f(x)dx$? (*Hint*: If you're stumped, try using a function that satisfies those properties, like x^2 .)

Matrix Algebra

Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

1. What is A times the first column of B ?
2. What is the transpose of A ? (Commonly written as A^T or A' .)
3. What is AB ?
4. Why is B not invertible?
5. If X is a 10-by-4 matrix, and y is a 4-by-1 vector, then which of Xy and yX is valid, and what are the dimensions of the result?
6. What does it mean for two vectors to be *orthogonal*? How do you test if two vectors are orthogonal?

Basic Programming

This section is to test concepts, not syntax knowledge of R or any other language.

1. What is the difference between = and == in the following code?

```
for (x in [1, 2, 3, 4]) {  
  y = x * x  
  if (y == 9) print("y is 9")  
}
```

2. Do you understand the difference between a programming variable (like y in the above code) and a mathematical variable, (like y in the equation $y = x^2$).
3. Can you write a function (in any programming language, or in “pseudo-code”) that takes an integer n as input and outputs n factorial. Factorial is defined as $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 1$. (Use a for loop or a while loop and basic mathematical operations.)

Thank you for completing this self-test. If you are comfortable with the level of math skills in these questions, and you understand that you are expected to know this information when you start the Certificate, **please add this statement to your admissions letter, "I have completed the Math Skills Self-Test."** If you cannot answer all these questions, or are uncertain of your answers, we recommend completing the following resources:

- Logarithms: Khan Academy, Logarithms
<https://www.khanacademy.org/math/algebra2/logarithms-tutorial>
- Calculus:
 - Derivatives: Khan Academy, Differential calculus, especially Power Rule and Chain Rule sections
<https://www.khanacademy.org/math/differential-calculus/taking-derivatives>
 - Integrals: Khan Academy, Integrals
<https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals>
- Matrix Algebra: Stat Trek, Matrix Algebra Tutorial, complete through the *Matrix inverse* section
<http://stattrek.com/tutorials/matrix-algebra-tutorial.aspx>
- Programming: A (very) short introduction to R
<http://cran.r-project.org/doc/contrib/Torfs+Brauer-Short-R-Intro.pdf>

Answers:

Logarithms and Algebra (1) 5; (2) -3; (3) intercept is -1, slope is 3; (4) intercept is $\log 4$, slope is -1; (5) $2ab$; (6) 10.

Calculus (1) Find the value(s) of x that make $\frac{d}{dx}f = 0$. If the second derivative is negative, it is a maximum. (2) $6e^{2x+1}$. (3) $4/x - 6/(1-x)$. (4) $\frac{1}{2}$. (5) 0.

Matrix Algebra (1) $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$; (2) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$; (3) $\begin{pmatrix} 7 & 14 \\ 4 & 8 \end{pmatrix}$; (4) So many reasons (all equivalent): the second column is a multiple of the first; the second row is a multiple of the first; the determinant is 0; I tried every single

possible 2-by-2 matrix and none of them worked. (5) Xy is a 10-by-1 vector. (6) Orthogonal vectors have a 90° angle between them, their dot-product is 0.

Computer Programming (1) The = is being used for *assignment*, associating a value with a variable, the == is a test of equality which will be True or False. (2) A programming variable, y , takes a single value at a time (or could possibly be assigned to be something other than a number: a string, an array, a matrix. . .); it may be assigned and then re-assigned to something completely different. A mathematical variable, y , is meaningless in the absence of an equation; it can also represent a single number or a matrix or a function, but it doesn't necessarily take on a single value. (3) Two possibilities:

```
factorial = function(n) {
  resultt = 1
  i = 2
  while (i <= n) {
    result = result * i
    i = i + 1
  }
  return(result)
}
```

```
factorial = function(n) {
  result = n
  for (i in (n - 1):1) {
    result = result * i
  }
  return(result)
}
```